

# Scheduling tasks using combinatorial auctions: the MAGNET approach<sup>\*</sup>

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**Abstract.** We consider the problem of rational, self-interested, economic agents who must negotiate with each other in a market environment in order to carry out their plans. Customer agents express their plans in the form of task networks with temporal and precedence constraints. A combinatorial reverse auction allows supplier agents to submit bids specifying prices for combinations of tasks, along with time windows and duration data that the customer may use to compose a work schedule. We describe the consequences of allowing the advertised task network to contain schedule infeasibilities, and show how to resolve them in the auction winner-determination process.

## 1 Introduction

We believe that much of the commercial potential of the Internet will remain unrealized until a new generation of autonomous systems is developed and deployed. A major problem is that the global connectivity and rapid communication capabilities of the Internet can present an organization with vast numbers of alternative choices, to the point that users are overwhelmed, and conventional automation is insufficient.

Much has been done to enable simple buying and selling over the Internet, and systems exist to help customers and suppliers find each other, such as search engines, vertical industry portals, personalization systems, and recommender engines. However, many business operations are much more complex than the simple buying and selling of individual items. We are interested in situations that require coordinated combinations of goods and services, where there is often some sort of constraint-satisfaction or combinatorial optimization problem that needs to be solved in order to assemble a “deal.” Commonly, these extra complications are related to constraints among task and services, and to time limitations. The combinatorics of such situations are not a major problem when an organization is working with small numbers of partners, but can easily become nearly insurmountable when “opened up” to the public Internet.

We envision a new generation of systems that will help organizations and individuals find and exploit opportunities that are otherwise inaccessible or too complex to seriously evaluate. These systems will help potential partners find

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each other (matchmaking), negotiate mutually beneficial deals (negotiation, evaluation, commitment), and help them monitor the progress of distributed activities (monitoring, dispute resolution). They will operate with variable levels of autonomy, allowing their principals (users) to delegate or reserve authority as needed, and they will provide their principals with a market presence and power that is far beyond what is currently achievable with today's telephone, fax, web, and email-based methods. We believe that an important negotiation paradigm among these systems will be market-based combinatorial auctions, with added precedence and temporal constraints.

The Multi-AGent NEgotiation Testbed (MAGNET) project represents a first step in bringing this vision to reality. MAGNET provides a unique capability that allows self-interested agents to negotiate over complex coordinated tasks, with precedence and time constraints, in an auction-based market environment. This paper introduces many of the problems a customer agent must solve in the MAGNET environment and explores in detail the problem of solving the extended combinatorial-auction winner determination problem.

**Guide to this paper** Section 2 works through a complete interaction scenario with an example problem, describing each of the decision processes a customer agent must implement in order to maximize the expected utility of its principal. For some of them, we have worked out how to implement the decisions, while for the remainder we only describe the problems. Section 3 focuses on one specific decision problem, that of deciding the winners in a MAGNET auction. A number of approaches are possible; we describe an optimal tree search algorithm for this problem. Section 4 places this work in context with other work in the field. In particular, we draw on work in multi-agent negotiation, auction mechanism design, and combinatorial auction winner-determination, which has been a very active field in recent years. Finally, Section 5 wraps up the discussion and points out a set of additional research topics that must be addressed to further realize the MAGNET vision.

## 2 Decision processes in a MAGNET customer agent

We focus on negotiation scenarios in which the object of the interaction is to gain agreement on the performance of a set of coordinated tasks that one of the agents has been asked to complete. We assume that self-interested agents will cooperate in such a scheme to the extent that they believe it will be profitable for them to do so. After a brief high-level overview of the MAGNET system, we focus on the decision processes that must be implemented by an agent that acts as a customer in the MAGNET environment. We intend that our agents exhibit rational economic behavior. In other words, an agent should always act to maximize the *expected utility* of its principal.

We will use an example to work through the agent's decisions. Imagine that you own a small vineyard, and that you need to get last autumn's batch of wine

bottled and shipped<sup>1</sup>. During the peak bottling season, there is often a shortage of supplies and equipment, and your small operation must lease the equipment and bring on seasonal labor to complete the process. If the wine is to be sold immediately, then labels and cases must also be procured, and shipping resources must be booked. Experience shows that during the Christmas season, wine cases are often in short supply and shipping resources overbooked.

## 2.1 Agents and their environment

Agents may fulfill one or both of two roles with respect to the overall MAGNET architecture, as shown in Figure 1. A *Customer agent* pursues its goals by formulating and presenting *Requests for Quotations* (RFQs) to *Supplier agents* through a market infrastructure [1]. An RFQ specifies a task network that includes task descriptions, a precedence network, and temporal constraints that limit task start and completion times. Customer agents attempt to satisfy their goals for the greatest expected profit, and so they will accept bids at the least net cost, where cost factors can include not only bid prices, but also goal completion time, risk factors, and possibly other factors, such as preferences for specific suppliers. More precisely, these agents are attempting to maximize the utility function of some user, as discussed in detail in [2].

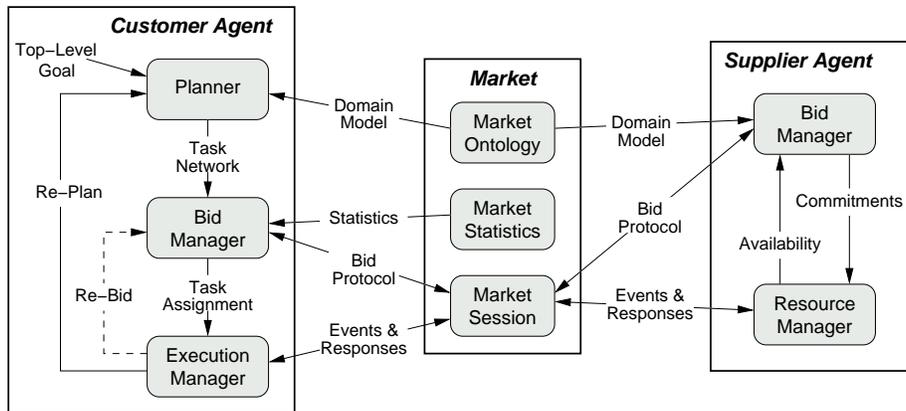


Fig. 1. The MAGNET architecture

A supplier agent attempts to maximize the value of the resources under its control by submitting bids in response to customer RFQs. A bid specifies what tasks the supplier is able to undertake, when it is available to perform those tasks, how long they will take to complete, and a price. Each bid may specify one or

<sup>1</sup> This example is taken from the operations of the Weingut W. Ketter winery, Kröv, Germany.

more tasks. Suppliers may submit multiple bids to specify different combinations of tasks, or possibly different time constraints with different prices. For example, a supplier might specify a short duration for some task that requires use of high cost overtime labor, as well as a longer duration at a lower cost using straight-time labor. MAGNET currently supports simple disjunction semantics for bids from the same supplier. This means that if a supplier submits multiple bids, any non-conflicting subset can be accepted. Other bid semantics are possible [3, 4].

## 2.2 Planning

A transaction (or possibly a series of transactions) starts when the agent is given a goal that must be satisfied. Attributes of the goal might include a payoff and a deadline, or a payoff function that varies over time.

While it would certainly be possible to integrate a general-purpose planning capability into a MAGNET agent, we expect that in many realistic situations the principal will already have a plan, perhaps based on standard industry practices. Figure 2 shows such a plan, for our winery bottling operation. We shall use this plan to illustrate the decision processes the agent must perform.

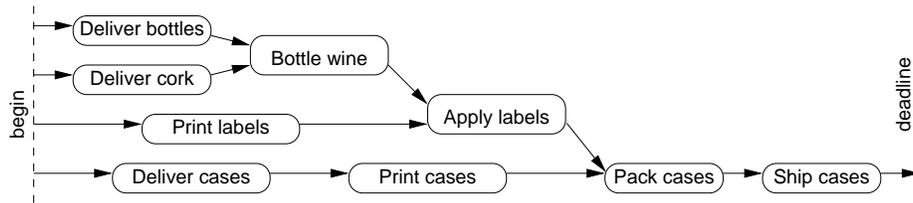


Fig. 2. Plan for the wine-bottling example.

Formally, we define a plan  $\mathcal{P} = (\mathcal{S}, \mathcal{V})$  as a task network containing a set of tasks  $\mathcal{S}$ , and a set of precedence relations  $\mathcal{V}$ . A precedence relation relates two tasks  $s, s' \in \mathcal{S}$  as  $s \prec s'$ , interpreted as “task  $s$  must be completed before task  $s'$  can start.”

We assume that markets will be sponsored by trade associations and commercial entities, and will therefore be more or less specialized. A consequence of this is that agents must in general deal in multiple markets to accomplish their goals. For our example, we assume that the tasks in our plan are associated with markets as specified in Table 1.

It appears that we will need to deal with 3 different markets, and we will pack the cases ourselves. Or perhaps we’ll open a few bottles and invite the village to help out.

So far, our plan is not situated in time, and we have not discussed our expected payoff for completing this plan. In the wine business, the quality and value of the product depends strongly on time. The wine must be removed from the casks within a 2-week window, and the bottling must be done immediately.

**Table 1.** Tasks and market associations for the wine-bottling example

Task	Description	Market
$s_1$	Deliver bottles	Vineyard Services
$s_2$	Deliver cork	Vineyard Services
$s_3$	Bottle wine	Vineyard Services
$s_4$	Print labels	Printing & Graphic Arts
$s_5$	Apply labels	Vineyard Services
$s_6$	Print cases	Vineyard Services
$s_7$	Deliver cases	Vineyard Services
$s_8$	Pack cases	(none)
$s_9$	Ship cases	Transport Services

For some varieties, the price we can get for our wine is higher if we can ship earlier, given a certain quality level. All the small vineyards in the region are on roughly the same schedule, so competition for resources during the prime bottling period can be intense. Without specifying the exact functions, we assume that the payoff drops off dramatically if we miss the 2-week bottling window, and less dramatically as the shipment date recedes into the future.

This example is of course simplified to demonstrate our ideas. For example, we are treating the bottling and labeling operations as atomic – the entire bottling operation must be finished before we can start labeling – even though common-sense would inform us that we would probably want to apply this constraint to individual bottles, rather than to the entire batch. On the other hand, some varieties of wine are aged in the bottles for 6 months or more before the labels are applied.

### 2.3 Planning the bidding process

At this point, the agent has a plan, and it knows which markets it must deal in to complete the plan. It also knows the value of completing the plan, and how that value depends on time. The next step is to decide how best to use the markets to maximize its utility. It will do this in two phases. First, the agent generates an overall plan for the bidding process, which may involve multiple RFQs in each of multiple markets. We call this a “bid-process plan”. Then a detailed timeline is generated for each RFQ.

The simplest bid-process plan would be to issue a single RFQ in each market, each consisting of the portion of the plan that is relevant to its respective market. If all RFQs are issued simultaneously, and if they are all on the same timeline, then we can combine their bids and solve the combined winner-determination problem in a single step. However, this might not be the optimum strategy. For example:

- We may not have space available to store the cases if we are not ready to pack them when they arrive.

- Our labor costs might be much lower if we can label as we bottle; otherwise, we will need to move the bottles into storage as we bottle, then take them back out to label them.
- Once cases are packed, it is easy for us to store them for a short period. This means that we can allow some slack between the packing and shipping tasks.
- There is a limit to what we are willing to pay to bottle our wine, and there is a limit to the premium we are willing to pay to have the bottling completed earlier.

The agent can represent these issues as additional constraints on the plan, or in some cases as alternative plan components. For example, we could constrain the interval between  $s_5$  (labeling) and  $s_8$  (packing) to a maximum of one day, or we could add an additional storage task between  $s_3$  (bottling) and  $s_5$  that must be performed just in case there is a non-zero delay between the end of  $s_3$  and the start of  $s_5$ .

There are many possible alternative actions that the agent can take to deal with these issues. It need not issue RFQs in all markets simultaneously. It need not include all tasks for a given market in a single RFQ. Indeed, dividing the plan into multiple RFQs can be an important way to reduce scheduling uncertainty. For example, we might want to have a firm completion date for the bottling and labeling steps before we order the cases. When a plan is divided into multiple RFQs that are negotiated sequentially, then the results of the first negotiation provide additional constraints on subsequent negotiations.

Market statistics can be used to support these decisions. For example, if we knew that resources were readily available for the steps up through the labeling process (tasks  $s_1 \dots s_5$ ), we could include the case delivery and printing steps (tasks  $s_6$  and  $s_7$ ) in the same RFQ. This could be advantageous if suppliers were more likely to bid or likely to bid lower prices if they could bid on more of the business in a single interaction. In other words, some suppliers might be willing to offer a discount if we agree to purchase both bottles and cases from them, but if we negotiate these two steps in separate RFQs, we eliminate the ability to find out about such discounts.

We should note that suppliers can either help or hinder the customer in this process, depending on the supplier's motivations. For example, the supplier can help the customer mitigate issues like the constraint between bottling and packing. For example, if a supplier knew about this constraint, it could offer both tasks at appropriate times, or it could give the customer the needed scheduling flexibility by offering the case delivery over a broad time window or with multiple bids with a range of time windows. In some domains this could result in higher costs, due to the large speculative resource reservations the supplier would have to commit to in order to support its bids. On the other hand, if a supplier saw an RFQ consisting of  $s_6$  and  $s_7$ , it would know that the customer had likely already made commitments for the earlier tasks, since nobody wants cases printed if they aren't bottling. If the supplier also knew that there would be little competition

within the customer’s specified time window, it could inflate its prices, knowing that the customer would have little choice.

The bid-process plan that results from this decision process is a network of negotiation tasks and decision points. Figure 3 shows a possible bid-process plan for our wine-bottling example.

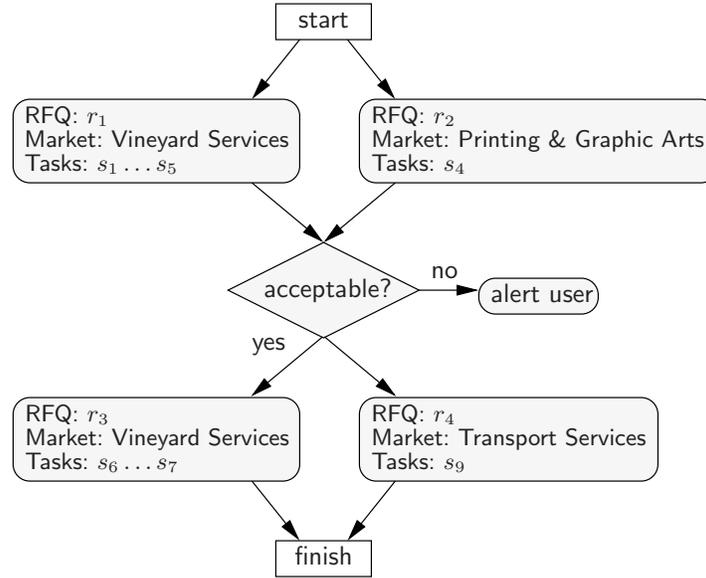
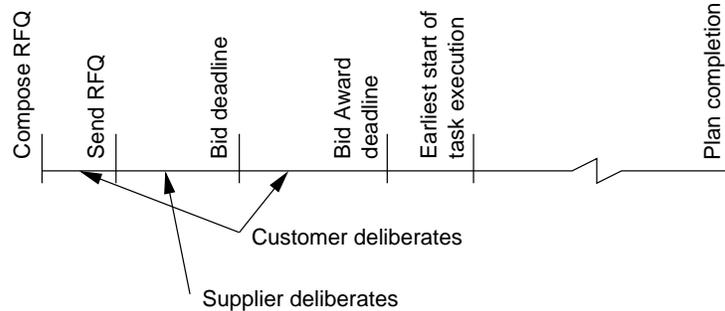


Fig. 3. Bid-process plan for the wine-bottling example.

Once we have a bid-process plan, we know what markets we will operate in, and how we want to divide up the bidding process. We must then schedule the bid-process plan, and allocate time within each RFQ/bidding interaction. These two scheduling problems may need to be solved together if the bid-process plan contains multiple steps and it is important to finish it in minimum time. Each RFQ step needs to start at a particular time, or when a particular event occurs or some condition becomes true. For example, if the rules of the market require deposits to be paid when bids are awarded, the customer may be motivated to move RFQ steps as late as possible, other factors being equal. On the other hand, if resources such as our bottling and labeling steps are expected to be in short supply, the agent may wish to gain commitments for them as early as possible in order to optimize its own schedule and payoff. We assume these decisions can be supported by market statistics, the agent’s own experience, and/or the agent’s principal.

Each RFQ must also be allocated enough time to cover the necessary deliberation processes on both the customer and supplier sides. Some of these processes may be automated, and some may involve user interaction. The timeline in Fig-

ure 4 shows an abstract view of the progress of a single negotiation. At the beginning of the process, the customer agent must allocate deliberation time to itself to compose its RFQ<sup>2</sup>, to the supplier for bid preparation, and to itself again for the bid evaluation process. Two of these time points, the bid deadline and the bid award deadline, must be communicated to suppliers as part of the RFQ. The bid deadline is the latest time a supplier may submit a bid, and the bid award deadline is the earliest time a supplier may expire a bid. The interval between these two times is available to the customer to determine the winners of the auction.



**Fig. 4.** Typical timeline for a single RFQ

In general, it is expected that bid prices will be lower if suppliers have more time to prepare bids, and more time and schedule flexibility in the execution phase. Minimizing the delay between the bid deadline and the award deadline will also minimize the supplier’s opportunity cost, and would therefore be expected to reduce bid prices. On the other hand, the customer’s ability to find a good set of bids is dependent on the time allocated to bid evaluation, and if a user is making the final decision on bid awards, she may want to run multiple bid-evaluation cycles with some additional think time. We are interested in the performance of the winner determination process precisely because it takes place within a window of time that must be determined ahead of time, before bids are received, and because we expect better overall results, in terms of maximizing the agent’s utility, if we can maximize the amount of time available to suppliers while minimizing the time required for customer deliberation. These time intervals can be overlapped to some extent, but doing so can create opportunities for strategic manipulation of the customer by the suppliers, as discussed in [6].

The process for setting these time intervals could be handled as a non-linear optimization problem, although it may be necessary to settle for an approximation. This could consist of estimating the minimum time required for the

<sup>2</sup> This may be a significant combinatorial problem – see for example [5].

customer’s processes, and allocating the remainder of the available time to the suppliers, up to some reasonable limit.

## 2.4 Composing a request for quotes

At this point in the agent’s decision process, we have the information needed to compose one or more RFQs, we know when to submit them, and we presumably know what to do if they fail (if we fail to receive a bid set that covers all the task in the RFQ, for example). The next step is to set the time windows for tasks in the individual RFQs, and submit them to their respective markets.

Formally, an RFQ  $r = (\mathcal{S}_r, \mathcal{V}_r, \mathcal{W}_r, \tau)$  contains a subset  $\mathcal{S}_r$  of the tasks in the task network  $\mathcal{P}$ , with their precedence relations  $\mathcal{V}_r$ , the task time windows  $\mathcal{W}_r$  specifying constraints on when each task may be started and completed, and the RFQ timeline  $\tau$  containing at least the bid deadline and bid award deadline. As we discussed earlier, there might be elements of the task network  $\mathcal{P}$  that are not included in the RFQ. For each task  $s \in \mathcal{S}_r$ , the RFQ we must specify a time window  $w \in \mathcal{W}_r$ , consisting of an earliest start time  $t_{es}(s, r)$  and a latest finish time  $t_{lf}(s, r)$ , and a set of precedence relationships  $\mathcal{V}_r = \{s' \in \mathcal{S}_r, s' \prec s\}$ , associating  $s$  with each of the other tasks  $s' \in \mathcal{S}_r$  whose completion must precede the start of  $s$ .

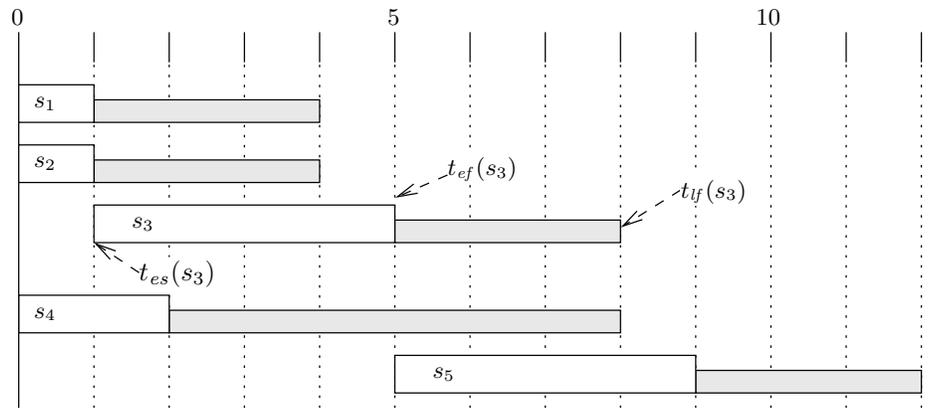
The principal outcome of the RFQ-generation process is a set of values for the early-start and late-finish times for the time windows  $\mathcal{W}_r$  in the RFQ. We obtain a first approximation using the Critical Path (CPM) algorithm [7], after making some assumptions about the durations of tasks, and about the earliest start time for tasks that have no predecessors in the RFQ (the *root tasks*  $\mathcal{S}_R$ ) and the latest finish times for tasks that have no successors in the RFQ (the *leaf tasks*  $\mathcal{S}_L$ ). Market mean-duration statistics could be used for the task durations. Overall start and finish times for the tasks in the RFQ may come from the bid-process plan, or we may already have commitments that constrain them as a result of other activities. For this discussion, we assume a continuous-time domain, although we realize that many real domains effectively work on a discrete-time basis. Indeed, it is very likely that some of our wine bottling activities would typically be quoted in whole-day increments. We also ignore calendar issues such as overtime/straight time, weekends, holidays, time zones, etc.

The critical path algorithm walks the directed graph of tasks and precedence constraints, forward from the early-start times of the root tasks to compute the earliest start  $t_{es}(s)$  and finish  $t_{ef}(s)$  times for each task  $s \in \mathcal{S}_r$ , and then backward from the late-finish times of the leaf tasks to compute the latest finish  $t_{lf}(s)$  and start  $t_{ls}(s)$  times for each task. The minimum duration of the entire task network specified by the RFQ, defined as  $\max_{s' \in \mathcal{S}_L}(t_{ef}(s')) - \min_{s \in \mathcal{S}_R}(t_{es}(s))$ , is called the *makespan* of the task network. The smallest slack in any leaf task  $\min_{s \in \mathcal{S}_L}(t_{lf}(s) - t_{ef}(s))$  is called the *total slack* of the task network within the RFQ. All tasks  $s$  for which  $t_{lf}(s) - t_{ef}(s) = \text{total-slack}$  are called *critical tasks*. Paths in the graph through critical tasks are called *critical paths*.

Some situations will be more complex than this, such as the case when there are constraints that are not captured in the precedence network of the RFQ. For

example, some non-leaf task may have successors that are already committed but are outside the RFQ. The CPM algorithm is still applicable, but the definition of critical tasks and critical paths becomes more complex.

Figure 5 shows the result of running the CPM algorithm on the tasks of RFQ  $r_1$  from our bid-process plan. We are assuming task durations as given in the individual “task boxes.” We observe several problems immediately. The most obvious is that it is likely that many bids returned in response to this RFQ would conflict with one another because they would fail to combine feasibly. For example, if I had a bid for the label printing task  $s_4$  for days 5-7, then the only bids I could accept for the labeling task  $s_5$  would be those that had a late start time at least as late as day 7. If the bids for  $s_5$  were evenly distributed across the indicated time windows, and if all of them specified the same 4-day duration, then only 1/3 of those bids could be considered. In general, we want to allow time windows to overlap, but excessive overlap is almost certainly counterproductive. We will revisit this issue shortly.



**Fig. 5.** Initial time allocations for tasks in RFQ  $r_1$ . Only the  $t_{es}(s)$  and  $t_{lf}(s)$  times are actually specified in the RFQ.

Once we have initial estimates from the CPM algorithm, there are several issues to be resolved, as described in the following sections.

**Setting the total slack** The plan may have a hard deadline, which may be set by a user or determined by existing commitments for tasks that cannot be started until tasks in the current RFQ are complete. Otherwise, in the normal case, the bid-process plan is expected to set the time limits for the RFQ.

It would be interesting to find a way to use the market to dynamically derive a schedule that maximizes the customer’s payoff. This would require cooperation of bidders, and could be quite costly. Parkes and Ungar [8] have done something

like this in a restricted domain, but it's hard to see how to apply it to the more generalized MAGNET domain.

**Task ordering** For any pair of tasks in the plan that could potentially be executed in parallel, we may have a choice of handling them in parallel, or in either sequential order. For example, in our wine-bottling example, we could choose to acquire the bottles before buying the corks. In general, if there is uncertainty over the ability to complete tasks which could cause the plan to be abandoned, then (given some straightforward assumptions such as payments being due when work is completed) the agent's financial exposure can be affected by task ordering. If a risky task is scheduled ahead of a "safe" task, then if the risky task fails we can abandon the plan without having to pay for the safe task. Babanov [5] has worked out in detail how to use task completion probabilities and discount rates in an expected-utility framework to maximize the probabilistic "certain payoff" for an agent with a given risk-aversion coefficient.

For some tasks, linearizing the schedule will extend the plan's makespan, and this must be taken into account in terms of changes to the ultimate payoff. Note that in many cases the agent may have flexibility in the start time as well as the completion time of the schedule. This would presumably be true of our wine-bottling example.

**Allocating time to individual tasks** Once we have made decisions about the overall time available and about task ordering, the CPM algorithm gives us a set of preliminary time windows. In most cases, this will not produce the best results, for several reasons:

**Resource availability** – In most markets, services will vary in terms of availability and resource requirements. There may be only a few dozen portable bottling and labeling machines in the region, while corks may be stored in a warehouse ready for shipping. There is a high probability that one could receive several bids for delivery of corks on one specific day, but a much lower probability that one could find even one bid for a 6-day bottling job for a specific 6-day period. More likely one would have to allow some flexibility in the timing of the bottling operation in order to receive usable bids.

**Lead-time effects** – In many industries, suppliers have resources on the payroll that must be paid for whether their services are sold or not. In these cases, suppliers will typically attempt to "book" commitments for their resources into the future. In our example, the chances of finding a print shop to produce our labels tomorrow is probably much lower than the chances of finding shops to print them next month. This means that, at least for some types of services, one must allow more scheduling flexibility to attract short lead time bids than for longer lead times. We should also expect to pay more for shorter lead times.

**Task-duration variability** – Some services are very standardized (delivering corks, printing 5000 labels), while others may be highly variable, either because they rely on human creativity (software development) or the weather

(bridge construction), or because different suppliers use different processes, different equipment, or different staffing levels (wine bottling). These two types of variability can usually be differentiated by the level of predictability; suppliers that use a predictable process with variable staffing levels are likely to be able to deliver on time on a regular basis, while services that are inherently unpredictable will tend to exhibit frequent deviations from the predictions specified in bids<sup>3</sup>. For services that exhibit a high variability in duration, as specified in bids, the customer's strategy may depend on whether a large number of bidders is expected, and whether there is a correlation between bid price and quoted task duration. If a large number of bidders is expected, then the customer may be able to allocate a below-average time window to the task, in the expectation that there will be some suppliers at the lower end of the distribution who will be able to perform within the specified window. On the other hand, if few bidders are expected, a larger than average time window may be required in order to achieve a reasonable probability of receiving at least one usable bid.

**Excessive allocations to non-critical tasks** – One obvious problem with the time allocations from the CPM algorithm as shown in Figure 5 is that non-critical tasks (tasks not on the critical path) are allocated too much time, causing unnecessary overlap in their time windows. All other things being equal, we are likely to be better off if RFQ time windows do not overlap, because we will have fewer infeasible bid combinations.

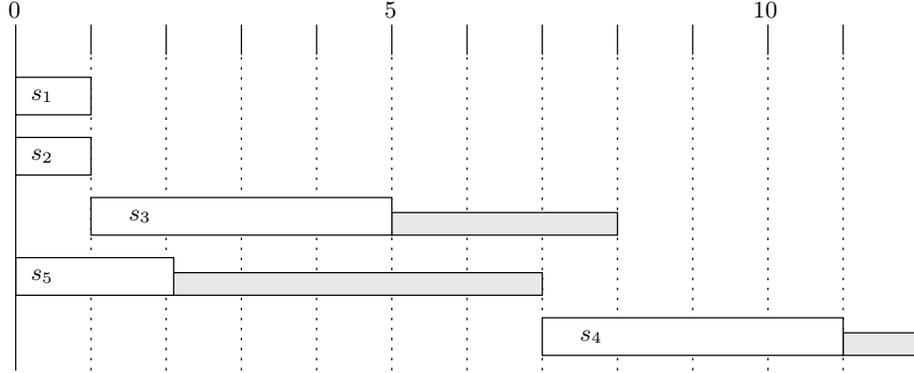
**Trading off feasibility for flexibility** In general we expect more bidders, and lower bid prices, if we offer suppliers more flexibility in scheduling their resources by specifying wider time windows. On the other hand, if we define RFQ time windows with excessive overlap, a significant proportion of bid combinations will be unusable due to schedule infeasibility. The intuition is that there will be some realistic market situations where the customer is better off allowing RFQ time windows to overlap to some degree, if we take into account price, plan completion time, and probability of successful plan completion (which requires at minimum a set of bids that covers the task set and can be composed into a feasible schedule). This means that the winner-determination procedure must handle schedule infeasibilities among bids.

Figure 6 shows a possible updated set of RFQ time windows for our wine-bottling example, taking into account the factors we have discussed. We have shortened the time windows for tasks  $s_1$  and  $s_2$ , because we believe that bottles and corks are readily available, and can be delivered when needed. There is no advantage to allowing more time for these tasks. Market data tells us that bottling services are somewhat more difficult to schedule than labeling services, and so we have specified a wider time window for task  $s_3$  than for  $s_4$ . Our deadline is such that the value of completing the work a day or two earlier is

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<sup>3</sup> Whether the market or customers would be able to observe these deviations may depend on market rules and incentives, such as whether a supplier can be paid early by delivering early.

higher than the potential loss of having to reject some conflicting bids. We also know from market data that a large fraction of suppliers of the bottling crews can also provide the labeling service, and so the risk of schedule infeasibility will be reduced if we receive bids for both bottling and labeling. Finally, there is plenty of time available for the non-critical label-printing task  $s_5$  without needing to overlap its time window with its successor task  $s_4$ .



**Fig. 6.** Revised time allocations for tasks in RFQ  $r_1$ .

## 2.5 Evaluating bids

Once an RFQ is issued and the bids are returned, the agent must decide which bids to accept. The bidding process is an extended combinatorial auction, because bids can specify multiple tasks, and there are additional constraints the bids must meet (the precedence constraints) other than just covering the tasks. The winner-determination process must choose a set of bids that maximize the agent’s utility, covers all tasks in the associated RFQ, and forms a feasible schedule.

**Formal description of the winner-determination problem** Each bid represents an offer to execute some subset of the tasks specified in an RFQ, for a specified price, within specified time windows. Formally, a bid  $b = (r, \mathcal{S}_b, \mathcal{W}_b, c_b)$  consists of a subset  $\mathcal{S}_b \in \mathcal{S}_r$  of the tasks specified in the corresponding RFQ  $r$ , a set of time windows  $\mathcal{W}_b$ , and an overall cost  $c_b$ . Each time window  $w_s \in \mathcal{W}_b$  specifies for a task  $s$  an earliest start time  $t_{es}(s, b)$ , a latest start time  $t_{ls}(s, b)$ , and a task duration  $d(s, b)$ .

It is a requirement of the protocol that the time window parameters in a bid  $b$  are within the time windows specified in the RFQ, or  $t_{es}(s, b) \geq t_{es}(s, r)$  and  $(t_{ls}(s, b) + d(s, b)) \leq t_{lf}(s, r)$  for a given task  $s$  and RFQ  $r$ . This requirement may be relaxed, although it is not clear why a supplier agent would want to

expose resource availability information beyond that required to respond to a particular bid. For bids that specify multiple tasks, it is also a requirement that the time windows in the bids be internally feasible. In other words, for any bid  $b$ , if for any two of its tasks  $(s_i, s_j) \in \mathcal{S}_b$  there is a precedence relation  $s_i \prec s_j$  specified in the RFQ, then it is required that  $t_{es}(s_i, b) + d(s_i, b) \leq t_{ls}(s_j, b)$ .

A solution to the bid-evaluation problem is defined as a complete mapping  $\mathcal{S} \rightarrow \mathcal{B}$  of tasks to bids in which each task in the corresponding RFQ is mapped to exactly one bid, and that is consistent with the temporal and precedence constraints on the tasks as expressed in the RFQ and the mapped bids.

Figure 7 shows a very small example of the problem the bid evaluator must solve. As noted before, there is scant availability of bottling equipment and crews, so we have provided an ample time window for that activity. At the same time, we have allowed some overlap between the bottling and labeling tasks, perhaps because we believed this would attract a large number of bidders with a wide variation in lead times and lower prices. Bid 1 indicates this bottling service is available from day 3 through day 7 only, and will take the full 5 days, but the price is very good. Similarly, bid 2 offers labeling from day 7 through day 10 only, again for a good price. Unfortunately, we can't use these two bids together because of the schedule infeasibility between them. Bid 3 offers bottling for any 3-day period from day 2 through day 7, at a higher price. We can use this bid with bid 2 if we start on day 4, but if we start earlier we will have to handle the unlabeled bottles somehow. Finally, bid 4 offers both the bottling and labeling services, but the price is higher and we would finish a day later than if we accepted bids 2 and 3.

**Evaluation criteria** We have discussed the winner-determination problem in terms of price, task coverage, and schedule feasibility. In many situations, there are other factors that can be at least as important as price. For example, we might know (although the agent might not know) that the bottling machine being offered in bid 3 is prone to breakdown, or that it tends to spill a lot of wine. We might have a long-term contract with one of the suppliers, Hermann, that gives us a good price on fertilizer only if we buy a certain quantity of corks from him every year. We might also know that one of the local printers tends to miss his time estimates on a regular basis, but his prices are often worth the hassle, as long as we build some slack into the schedule when we award a bid to him.

Of course, including these other factors will distort a “pure” auction market, since the lowest-price bidder will not always win. As a practical matter, such factors are commonly used to evaluate potential procurement decisions, and real market mechanisms must include them if they are to be widely acceptable.

Many of these additional factors can be expressed as additional constraints on the winner-determination problem, and some can be expressed as cost factors. These constraints can be as simple as “don't use bid  $b_3$ ” or more complex, as in “if Hermann bids on corks, and if a solution using his bid is no more than 10% more costly than a solution without his bid, then award the bid to Hermann.”

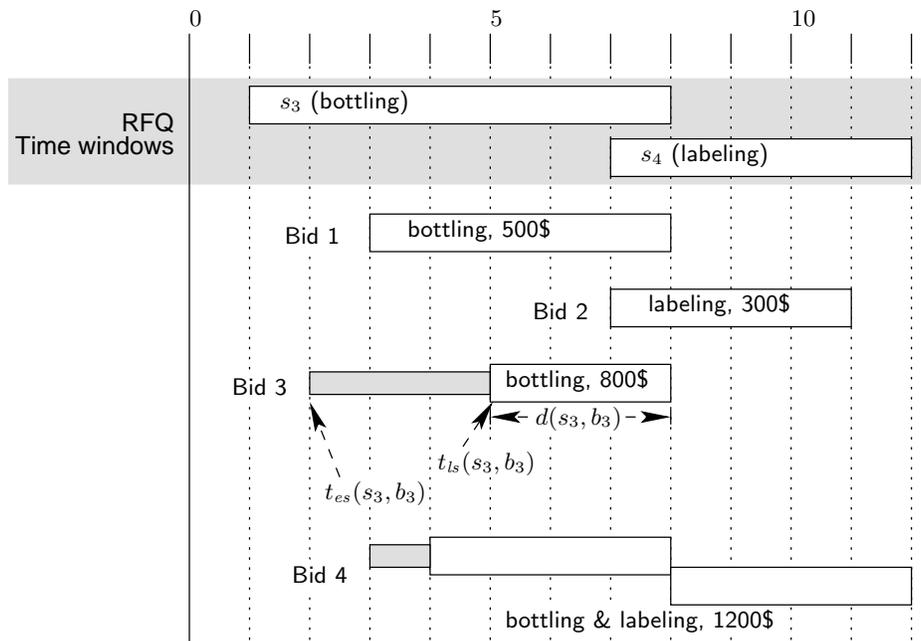


Fig. 7. Bid Example

Some of them can be handled by preprocessing, some must be handled within the winner-determination process, and some will require running it multiple times and comparing results.

**Mixed-initiative approaches** There are many environments in which an automated agent is unlikely to be given the authority to make unsupervised commitments on behalf a person or organization. In these situations, we expect that many of the decision processes we discuss here will be used as decision-support tools for a human decision-maker, rather than as elements of a completely autonomous agent. The decision to award bids is one that directly creates commitment, and so it is a prime candidate for user interaction. We have constructed an early prototype of such an interface. It allows a user to view bids, add simple bid inclusion and exclusion constraints, and run one of the winner-determination search methods. Bids may be graphically overlaid on the RFQ, and both the RFQ and bid time windows are displayed in contrasting colors on a Gantt-chart display.

Effective interactive use of the bid-evaluation functions of an agent require the ability to visualize the plan and bids, to visualize bids in groups with constraint violations highlighted, and to add and update constraints. The winner-determination solver must be accessible and its results presented in an understandable way, and there must be a capability to generate multiple alternative solutions and compare them.

## 2.6 Awarding bids

The result of the winner-determination process is a (possibly empty) mapping  $\mathcal{S} \rightarrow \mathcal{B}$  of tasks to bids. We assume that the bids in this mapping meet the criteria of the winner-determination process: they cover the tasks in the RFQ, can be composed into a feasible schedule, and they maximize the agent’s or user’s expected utility. However, we cannot just award the winning bids. In general, a bid  $b$  contains one or more offers of services for tasks  $s$ , each with a duration  $d(s, b)$  within a time window  $w(s, b) > d(s, b)$ . The price assumes that the customer will specify, as part of the bid award, a specific start time for each activity. Otherwise, the supplier would have to maintain its resource reservation until some indefinite future time when the customer would specify a start time. This would create a disincentive for suppliers to specify large time windows, raise prices, and complicate the customer’s scheduling problem.

This means that the customer must build a final work schedule before awarding bids. We will defer the issue of dealing with schedule changes as work progresses. This scheduling activity represents another opportunity to maximize the customer’s expected utility. In general, the customer’s utility at this point is maximized by appropriate distribution of slack in the schedule, and possibly also by deferring task execution in order to defer payment for completion.

## 3 Solving the MAGNET winner-determination problem

We now focus on the MAGNET winner-determination problem, originally introduced in Section 2.5. Earlier we have described both an Integer Programming formulation [9] and a simulated annealing framework for solving this problem [10] for this problem. In this chapter, we describe an application of the A\* method [11]. For simplicity, the algorithm presented here solves the winner-determination problem assuming that the payoff that does not depend on completion time.

The A\* algorithm is a method for finding optimal solutions to combinatorial problems that can be decomposed into a series of discrete steps. A classic example is finding the shortest route between two points in a road network. A\* works by constructing a tree of partial solutions. In general, tree search methods such as A\* are useful when the problem can be characterized by a solution path in a tree that starts at an initial node (root) and progresses through a series of expansions to a final node that meets the solution criteria. Each expansion generates successors (children) of some existing node, expansions continuing until a solution node is found. The questions of which node is chosen for expansion, and how the search tree is represented, lead to a family of related search methods. In the A\* method, the node chosen for expansion is the one with the “best” evaluation<sup>4</sup>, and the search tree is typically kept in memory in the form of a sorted queue. A\* uses an evaluation function

$$f(N) = g(N) + h(N) \tag{1}$$

---

<sup>4</sup> lowest for a minimization problem, highest for a maximization problem.

for a node  $N$ , where  $g(N)$  is the cost of the path from initial node  $N_0$  to node  $N$ , and  $h(N)$  is an estimate of the remaining cost to a solution node. If  $h(N)$  is a strict lower bound on the remaining cost (upper bound for a maximization problem), we call it an *admissible heuristic* and A\* is complete and optimal; that is, it is guaranteed to find a solution with the lowest evaluation, if any solutions exist, and it is guaranteed to terminate eventually if no solutions exist.

The winner determination problem for combinatorial auctions has been shown to be  $\mathcal{NP}$ -complete and inapproximable [12]. This result clearly applies to the MAGNET winner determination problem, since we simply apply an additional set of (temporal) constraints to the basic combinatorial auction problem, and we don't allow free disposal (because we want a set of bids that covers all tasks). In fact, because the additional constraints create additional bid-to-bid dependencies, and because bids can vary in both price and in time specifications, the bid-domination and partitioning methods used by others to simplify the problem (for example, see [13]) cannot be applied in the MAGNET case.

Sandholm has shown that there can be no polynomial-time solution, nor even a polynomial-time bounded approximation [13], so we must accept exponential complexity. We have shown in [14] that we can determine probability distributions for search time, based on problem size metrics, and we can use those empirically-determined distributions in our deliberation scheduling process.

Sandholm described an approach to solving the standard combinatorial auction winner-determination problem [13] using an iterative-deepening A\* formulation. Although many of his optimizations, such as the elimination of dominated bids and partitioning of the problem, cannot be easily applied to the MAGNET problem, we have adapted the basic structure of Sandholm's formulation, and we have improved upon it by specifying a means to minimize the mean branching factor in the generated search tree.

We describe a basic A\* formulation of the MAGNET winner-determination problem, and then we show how this formulation can be adapted to a depth-first iterative-deepening model [15] to reduce or eliminate memory limitations.

### 3.1 Bidtree framework

Our formulation depends on two structures which must be prepared before the search can run. The first is the *bidtree* introduced by Sandholm, and the second is the *bid-bucket*, a container for the set of bids that cover the same task set.

A bidtree is a binary tree that allows lookup of bids based on item content. The bidtree is used to determine the order in which bids are considered during the search, and to ensure that each bid combination is tested at most once. In Sandholm's formulation, the collection of bids into groups that cover the same item sets supports the discard of dominated bids, with the result that each leaf in the bidtree contains one bid. However, because our precedence constraints create dependencies among bids in different buckets, bid domination is a much more complex issue in the MAGNET problem domain. Therefore, we use bid-buckets at the leaves rather than individual bids.

The principal purpose of the bidtree is to support content-based lookup of bids. Suppose we have a plan  $\mathcal{S}$  with tasks  $s_m, m = 1..4$ . Further suppose that we have received a set of bids  $b_n, n = 1..10$ , with the following contents:  $b_1 : \{s_1, s_2\}$ ,  $b_2 : \{s_2, s_3\}$ ,  $b_3 : \{s_1, s_4\}$ ,  $b_4 : \{s_3, s_4\}$ ,  $b_5 : \{s_2\}$ ,  $b_6 : \{s_1, s_2, s_4\}$ ,  $b_7 : \{s_4\}$ ,  $b_8 : \{s_2, s_4\}$ ,  $b_9 : \{s_1, s_2\}$ ,  $b_{10} : \{s_2, s_4\}$ . Figure 8 shows a bidtree we might construct for this problem. Each node corresponds to a task. One branch, labeled *in*, leads to bids that include the task, and the other branch, labeled *out*, leads to bids that do not.

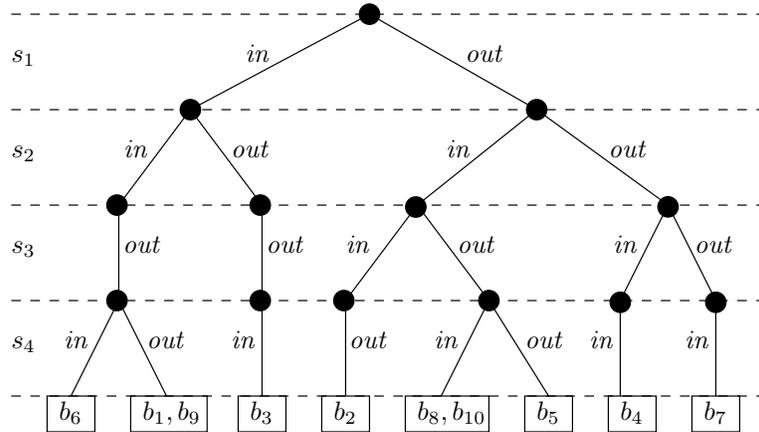


Fig. 8. Example bidtree, lexical task order

We use the bidtree by querying it for bid-buckets. A query consists of a *mask*, a vector of values whose successive entries correspond to the “levels” in the bidtree. Each entry in the vector may take on one of three values,  $\{in, out, any\}$ . A query is processed by walking the bidtree from its root as we traverse the vector. If an entry in the mask vector is *in*, then the *in* branch is taken at the corresponding level of the tree, similarly with *out*. If an entry is *any*, then both branches are taken at the corresponding level of the bidtree. So, for example, if we used a mask of  $[in, any, any, in]$ , the bidtree in Figure 8 would return the bid-buckets containing  $\{b_6\}$  and  $\{b_3\}$ .

A bid-bucket is a container for a set of bids that cover the same task set. In addition to the bid set, the bid-bucket structure stores the list of other bid-buckets whose bids conflict with its own (where we use “conflicts” to mean that they cover overlapping task sets). This recognizes the fact that all bids with the same task set will have the same conflict set.

In order to support computation of the heuristic function, we use a somewhat different problem formulation for A\* and IDA\* than we used for the IP formulation described in [9]. In that formulation, we were minimizing the sum of the costs of the selected bids. In this formulation, we minimize the cost of

each of the tasks, given a set of bid assignments. This allows for straightforward computation of the A\* heuristic function  $f(N)$  for a given node  $N$  in the search tree. We first define

$$f(N) = g(\mathcal{S}_m(N)) + h(\mathcal{S}_u(N)) \quad (2)$$

where  $\mathcal{S}_m(N)$  is the set of tasks that are mapped to bids in node  $N$ , while  $\mathcal{S}_u(N) = \mathcal{S}_r \setminus \mathcal{S}_m(N)$  is the set of tasks that are not mapped to any bids in the same node. We then define

$$g(\mathcal{S}_m(N)) = \sum_{j|s_j \in \mathcal{S}_m} \frac{c(b_j)}{n(b_j)} \quad (3)$$

where  $b_j$  is the bid mapped to task  $s_j$ ,  $c(b_j)$  is the total cost of  $b_j$ ,  $n(b_j)$  is the number of tasks in  $b_j$ , and

$$h(\mathcal{S}_u(N)) = \sum_{j|s_j \in \mathcal{S}_u} \frac{c(b_j^*)}{n(b_j^*)} \quad (4)$$

where  $b_j^*$  is the “usable” bid for task  $s_j$  that has the lowest cost/task. By “usable,” we mean that the bid  $b_j^*$  includes  $s_j$ , and does not conflict (in the sense of having overlapping task sets) with any of the bids  $b_j$  already mapped in node  $N$ .

Note that the definition of  $g(\mathcal{S}_m(N))$  can be expanded to include other factors, such as risk estimates or penalties for inadequate slack in the schedule, and these factors can be non-linear. The only requirement is that any such additional factor must *increase* the value of  $g(\mathcal{S}_m(N))$ , and not decrease it, because otherwise the admissibility of the heuristic  $h(\mathcal{S}_u(N))$  will be compromised, and we no longer would have an optimal search method.

### 3.2 A\* formulation

Now that we have described the bidtree and bid-bucket, we can explain our optimal tree search formulation. The algorithm is given in Figure 9.

The principal difference between this formulation and the “standard” A\* search formulation (see, for example, [11]), is that nodes are left on the queue (line 15) until they cannot be expanded further, and only a single expansion is tried (line 17) at each iteration. This is to avoid expending unnecessary effort evaluating nodes.

The expansion of a parent node  $N$  to produce a child node  $N'$  (line 17 in Figure 9) using the bidtree is shown in Figure 10. Here we see the reason to keep track of the buckets for the candidate-bid set of a node. In line 16, we use the mask for a new node to retrieve a set of bid-buckets. In line 18, we see that if the result is empty, or if there is some unallocated task for which no usable bid remains, we can go back to the parent node and just dump the whole bucket that contains the candidate we are testing.

In line 17 of Figure 10, we must find the minimum-cost “usable” bids for all unallocated tasks  $\mathcal{S}_u$  (tasks not in the union of the task sets of  $\mathcal{B}_{N'}$ ), as

```

1 Procedure A*_search
2 Inputs:
3    $\{\mathcal{S}, \mathcal{V}\}$ : the task network to be assigned
4    $\mathcal{B}$ : the set of bids, represented as a bidtree
5 Output:
6    $N_{opt}$ : the node having a mapping  $\mathcal{M}(N_{opt}) = \mathcal{S} \rightarrow \mathcal{B}$ 
           of tasks to bids with an optimal evaluation, if one exists
7 Process:
8    $Q \leftarrow$  priority_queue, sorted by node evaluation  $f(N)$ 
9    $N_0 \leftarrow$  empty node
10   $mask \leftarrow \{in, any, any, cdots\}$ 
11   $\mathcal{B}_{N_0}^c \leftarrow$  bidtree_query( $\mathcal{B}, mask$ )
           “ $\mathcal{B}_{N_0}^c$  is a set of bids (in the form of a set of bid buckets), containing
           the bids that can be used to expand  $N_0$ ”
12  insert( $Q, N_0$ )
13  loop
14    if empty( $Q$ ) then return failure
15     $N \leftarrow$  first( $Q$ )
16    if solution( $N$ ) then return  $N$ 
17     $N' \leftarrow$  astar_expand( $N$ ) “see Figure 10”
18    if  $N' = null$  then remove_front( $Q$ ) “Remove nodes that fail to expand”
19    else if feasible( $N'$ ) then insert( $Q, N'$ )

```

**Fig. 9.** Bidtree-based A\* search algorithm.

discussed earlier. One way (not necessarily the most efficient way) to find the set of usable bids is to query the bidtree using the mask that was generated in line 14, changing the single *in* entry to *any*. If there is any unallocated task that is not covered by some bid in the resulting set, then we can discard node  $N'$  because it cannot lead to a solution (line 22). Because all other bids in the same bidtree leaf node with the candidate bid  $b_x$  will produce the same bidtree mask and the same usable-bid set, we can also discard all other bids in that leaf node from the candidate set of the parent node  $N$ .

This implementation is very time-efficient but A\* fails to scale to large problems because of the need to keep in the queue all nodes that have not been fully expanded. Limiting the queue length destroys the optimality and completeness guarantees. Some improvement in memory usage can be achieved by setting an upper bound once the first solution is found in line 18 of Figure 10. Once an upper bound *flimit* exists, then any node  $N$  for which  $f(N) > flimit$  can be safely discarded, including nodes already on the queue. Unfortunately, this helps only on the margin; there will be a very small number of problems for which the resulting reduction in maximum queue size will be sufficient to convert a failed or incomplete search into a complete one. We address this in the next section.

One of the design decisions that must be made when implementing a bidtree-based search is how to order the tasks (or items, in the case of a standard

```

1 Procedure astar_expand
2 Inputs:
3    $N$ : the node to be expanded
4 Output:
5    $N'$ : a new node with exactly one additional bid, or null
6 Process:
7    $buckets \leftarrow \emptyset$ 
8   while  $buckets = \emptyset$  do
9     if  $\mathcal{B}_N^c = \emptyset$  then return null " $\mathcal{B}_N^c$  is set of candidate bids for node  $N$ "
10     $b_x \leftarrow \text{choose}(\mathcal{B}_N^c)$  "pick a bid from the set of candidates"
11     $\mathcal{B}_N^c \leftarrow \mathcal{B}_N^c - b_x$  "remove the chosen bid from the set"
12     $N' \leftarrow$  new node
13     $\mathcal{B}_{N'} \leftarrow \mathcal{B}_N + b_x$  " $\mathcal{B}_{N'}$  is the set of bids in node  $N'$ "
14     $\mathcal{S}_u \leftarrow \text{unallocated\_tasks}(N')$  "tasks not covered by any bid  $b \in \mathcal{B}_{N'}$ "
15     $mask \leftarrow \text{create\_mask}(\mathcal{B}_{N'}^c)$ 
        "for each task that is covered by a bid in  $\mathcal{B}_{N'}$ , set the corresponding
        "entry to out. Then find the first task in  $s \in \mathcal{S}_u$  (the task in  $\mathcal{S}_u$  with
        "the minimum index in the bidtree) and set its entry to in. Set the
        "remaining entries to any"
16     $buckets \leftarrow \text{bidtree\_query}(\mathcal{B}, mask)$ 
17     $\mathcal{B}_u \leftarrow \forall s \in \mathcal{S}_u, \text{minimum\_usable\_bid}(s)$  "see the narrative"
18    if  $(\text{solution}(N'))$ 
         $\vee ((buckets \neq \emptyset) \wedge (\neg \exists s \in \mathcal{S}_u | \text{minimum\_usable\_bid}(s) = \text{null}))$ 
19    then
20       $\mathcal{B}_{N'}^c \leftarrow buckets$  "candidates for  $N'$ "
21    else
22       $\text{remove}(\mathcal{B}_{N'}^c, \text{bucket}(b_x))$ 
        "all bids in the bucket containing  $b_x$  in node  $N$  will produce the
        "same mask and therefore an empty candidate set or a task that
        "cannot be covered by any usable bid"
23    end while
24     $g(N') \leftarrow \sum_{b \in \mathcal{B}_{N'}} c_b$ 
25     $h(N') \leftarrow \sum_{b \in \mathcal{B}_u} \text{avg\_cost}(b)$ 
26    return  $N'$ 

```

**Fig. 10.** Bidtree-based node-expansion algorithm.

combinatorial auction) when building the bidtree. It turns out that this decision can have a major impact on the size of the tree that must be searched, and therefore on performance and predictability. As we have shown in [16], the tasks should be ordered such that the tasks with higher numbers of bids come ahead of tasks with lower numbers of bids. This ordering is exploited in line 18 of Figure 10, where bid conflicts are detected.

### 3.3 Iterative Deepening A\*

Iterative Deepening A\* (IDA\*) [15] is a variant of A\* that uses the same two functions  $g$  and  $h$  in a depth-first search, and which keeps in memory only the current path from the root to a particular node. In each iteration of IDA\*, search depth is limited by a threshold value  $f_{limit}$  on the evaluation function  $f(N)$ . We show in Figure 11 a version of IDA\* that uses the same bidtree and node structure as the A\* algorithm. The recursive core of the algorithm is shown in Figure 12. This search algorithm uses the same node expansion algorithm as we used for the A\* search, shown in Figure 10 above.

```

1 Procedure IDA*_search
2 Inputs:
3    $\{\mathcal{S}, \mathcal{V}\}$ : the task network to be assigned
4    $\mathcal{B}$ : the set of bids, represented as a bidtree
5 Output:
6    $N_{opt}$ : the node having a mapping  $\mathcal{M}(N_{opt}) = \mathcal{S} \rightarrow \mathcal{B}$  of tasks to bids
           with an optimal evaluation, if one exists
7 Process:
8    $N_0 \leftarrow$  empty node
9    $g(N_0) \leftarrow 0$ 
10   $h(N_0) \leftarrow \sum_{s \in \mathcal{S}} \text{avg\_cost}(\text{minimum\_bid}(s))$ 
11   $f_{limit} \leftarrow f(N_0)$ 
12   $mask \leftarrow \{in, any, any, \dots\}$ 
13   $best\_node \leftarrow null$ 
14  while ( $best\_node = null$ )  $\wedge$  ( $f_{limit} \neq \infty$ ) do
15     $\mathcal{B}_{N_0}^c \leftarrow \text{bidtree\_query}(\mathcal{B}, mask)$ 
           “ $\mathcal{B}_{N_0}^c$  is a set of bids (in the form of a set of bid buckets),
           containing the bids that can be used to expand  $N_0$ . We have to
           repeat this for every iteration.”
16     $new\_limit \leftarrow \text{dfs\_contour}(N_0)$ 
17    if  $best\_node = null$  then
18       $f_{limit} \leftarrow \max(new\_limit, z \cdot f_{limit})$  “see narrative in this section”
19    end while
20  return  $best\_node$ 

```

**Fig. 11.** Bidtree-based Iterative Deepening A\* search algorithm: top level.

Complete solutions are detected in line 10 of Figure 12. Because we are doing a depth-first search with  $f(N) < f_{limit}$ , we have no way of knowing whether there might be another solution with a lower value of  $f(N)$ . But we do have an upper bound on the solution cost at this point, so whenever a solution is found, the value of  $f_{limit}$  can be updated in line 11 of Figure 12. This follows the usage in Sandholm [13], and limits exploration to nodes (and solutions) that are better than the best solution found so far.

```

1 Procedure dfs_contour
2 Inputs:
3    $N$ : a node
4 Output:
5    $new\_limit$ : a candidate for the  $f_{limit}$  value for the next contour. This is
                    either the first  $f(N)$  value seen that is larger than  $f_{limit}$ , or
                    the value of the best solution node found. If it is determined
                    that no solution is possible, then we return  $\infty$ .
6 Process:
7    $new\_limit \leftarrow f(N)$ 
8   if  $new\_limit > f_{limit}$  then “enforce contour limit”
9     return  $new\_limit$ 
10  if  $solution(N)$  then “switch to branch-and-bound”
11     $f_{limit} \leftarrow new\_limit$  “set new upper bound”
12     $best\_node \leftarrow N$ 
13    return  $new\_limit$ 
14   $nextL \leftarrow \infty$ 
15  while  $\mathcal{B}_N^c \neq \emptyset$  do
16     $N' \leftarrow \text{astar\_expand}(N)$ 
17    if  $(N' \neq null) \wedge (feasible(N'))$  then
18       $new\_limit \leftarrow \text{dfs\_contour}(N')$ 
19       $nextL \leftarrow \min(nextL, new\_limit)$ 
20  end while
21  return  $nextL$ 

```

**Fig. 12.** Bidtree-based Iterative Deepening A\* search algorithm: depth-first contour.

Nodes are tested for feasibility in line 17 of Figure 12 to prevent consideration and further expansion of nodes that cannot possibly lead to a solution.

There is a single tuning parameter  $z$  shown in line 19 of Figure 11, which must be a positive number  $> 1$ . This controls the amount of additional depth explored in each iteration of the main loop that starts on line 15. If  $z$  is too small, then `dfs_contour()` is called repeatedly to expand essentially the same portion of the search tree, and progress toward a solution is slow. On the other hand, if  $z$  is too large, large portions of the search tree leading to suboptimal solutions will be explored unnecessarily. In general, more effective heuristic functions (functions  $h(N)$  that are more accurate estimates of remaining cost) lead to lower values of  $z$ . Experimentation using the heuristic shown in Equation 4 shows that a good value is  $z = 1.15$ , and that it is only moderately sensitive (performance falls off noticeably with  $z < 1.1$  or  $z > 1.2$ ).

## 4 Related Work

This work draws from several fields. In Computer Science, it is related to work in artificial intelligence and autonomous agents. In Economics, it draws from

auction theory and expected utility theory. From Operations Research, we draw from work in combinatorial optimization.

#### 4.1 Multi-agent negotiation

MAGNET proposes using an auction paradigm to support problem-solving interactions among autonomous, self-interested, heterogeneous agents. Several other approaches to multi-agent problem-solving have been proposed. Some of them use a “market” abstraction, and some do not.

Rosenschein and Zlotkin [17] show how the behavior of agents can be influenced by the set of rules system designers choose for their agents’ environment. In their study the agents are homogeneous and there are no side payments. In other words, the goal is to share the work, in a more or less “equitable” fashion, but not to have agents pay other agents for work. They also assume that each agent has sufficient resources to handle all the tasks, while we assume the contrary.

In Sandholm’s TRACONET system [18, 19], agents redistribute work among themselves using a contracting mechanism. Sandholm considers agreements involving explicit payments, but he also assumes that the agents are homogeneous – they have equivalent capabilities, and any agent can handle any task. MAGNET agents are heterogeneous, and in general do not have the resources or capabilities to carry out the tasks necessary to meet their own goals without assistance from others.

Both Pollack’s DIPART system [20] and the Multiagent Planning architecture (MPA) [21] assume multiple agents that operate independently. However, in both of those systems the agents are explicitly cooperative, and all work toward the achievement of a shared goal. MAGNET agents are trying to achieve their own goals and to maximize their own profits; there is no global or shared goal.

**Solving problems using markets and auctions** MAGNET uses an auction-based negotiation style because auctions have the right economic and motivational properties to support “reasonable” resource allocations among heterogeneous, self-interested agents. However, MAGNET uses the auction approach not only to allocate resources, but also to solve constrained scheduling problems.

A set of auction-based protocols for decentralized resource-allocation and scheduling problems is proposed in [22]. The analysis assumes that the items in the market are individual discrete time slots for a single resource, although there is a brief analysis of the use of the Generalized Vickrey Auctions [23] to allow for combinatorial bidding. A combinatorial auction mechanism for dynamic creation of supply chains was proposed and analyzed in [24]. This system deals with the constraints that are represented by a multi-level supply-chain graph, but does not deal with temporal and precedence constraints among tasks. MAGNET agents

must deal with multiple resources and continuous time, but we do not currently deal explicitly with multi-level supply chains<sup>5</sup>

Several proposed bidding languages for combinatorial auctions allow bidders to express constraints, for example [25, 4]. However, these approaches only allow bidders to communicate constraints to the bid-taker (suppliers to the customer, in the MAGNET scenario), while MAGNET needs to communicate constraints in both directions.

**Infrastructure support for negotiation** Markets play an essential role in the economy [26], and market-based architectures are a popular choice for multiple agents (see, for instance, [27–30] and our own MAGMA architecture [31]). Most market architectures limit the interactions of agents to manual negotiations, direct agent-to-agent negotiation [19, 32], or some form of auction [33]. The Michigan Internet AuctionBot [33] is a very interesting system, in that it is highly configurable, able to handle a wide variety of auction rules. It is the basis for the ongoing Trading Agent Competition [34], which has stimulated interesting research on bidding behavior in autonomous agents, such as [35].

Matchmaking, the process of making connections among agents that request services and agents that provide services, will be an important issue in a large community of MAGNET agents. The process is usually done using one or more intermediaries, called middle-agents [36]. Sycara et al. [37] present a language that can be used by agents to describe their capabilities and algorithms to use it for matching agents over the Web. Our system casts the Market in the role of matchmaker.

The MAGNET market infrastructure depends on an Ontology to describe services that can be traded and the terms of discourse among agents. There has been considerable attention to development of detailed ontologies for describing business and industrial domains [38–40].

## 4.2 Combinatorial auctions

Determining the winners of a combinatorial auction [41] is an  $\mathcal{NP}$ -complete problem, equivalent to the weighted bin-packing problem. A good overview of the problem and approaches to solving it is [42]. Dynamic programming [43] works well for small sets of bids, but it does not scale well, and it imposes significant restrictions on the bids. Sandholm [13, 44] relaxes some of the restrictions and presents an algorithm for optimal selection of combinatorial bids, but his bids specify only a price and a set of items. Hoos and Boutilier[45] describe a stochastic local search approach to solving combinatorial auctions, and characterize its performance with a focus on time-limited situations. A key element of their approach involves ranking bids according to expected revenue; it's very hard to see how this could be adapted to the MAGNET domain with temporal

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<sup>5</sup> Individual MAGNET agents can deal with multi-level supply chains by subcontracting, but this requires that the initial time allocation provide sufficient slack for the extra negotiation cycles.

and precedence constraints, and without free disposal<sup>6</sup>. Andersson et al. [46] describe an Integer Programming approach to the winner determination problem in combinatorial auctions. Nisan [25] extends this model to handle richer bidding languages for combinatorial auctions, and we have extended it to handle the MAGNET situation in [9]. More recently, Sandholm [44] has described an improved winner-determination algorithm called BOB that uses a combination of linear programming and branch-and-bound techniques. It is not clear how this technique could be extended to deal with the temporal constraints in the MAGNET problem, although the bid-graph structure may be of value.

One of the problems with combinatorial auctions is that they are nearly always run in a single round sealed-bid format, and this is the format MAGNET uses. Parkes and Ungar [47] have shown how to organize multiple-round combinatorial auctions. Another problem is that the individual items in a combinatorial auction are individual items; there is no notion of quantity. MAGNET will eventually need to address this. This limitation is overcome in [48] for simple items without side constraints. The addition of precedence constraints would seriously complicate their procedure, and it has not yet been attempted.

### 4.3 Deliberation scheduling

The principal reason we are interested in search performance is because the search is embedded in a real-time negotiation scenario, and time must be allocated to it before bids are received, and therefore before the exact dimensions of the problem are known. In [49], deliberation scheduling is done with the aid of anytime and contract algorithms, and performance profiles. An anytime algorithm is one that produces a continuously-improving result given additional time, and a contract algorithm is one that produces a result of a given quality level in a given amount of time, but may not improve given additional time. The best winner-determination algorithms we know of for the MAGNET problem have marginal anytime characteristics, and we know of no applicable contract-type algorithms. In fact, [13] presents an inapproximability result for the winner-determination problem, leading us to believe that there may not be an acceptable contract algorithm.

One way to think about deliberation scheduling is to assign the time required for deliberation a cost, and then to balance the cost of deliberation against the expected benefit to be gained by the results of the deliberation. This is the approach taken in [50]. However, much of this analysis assumes that there is a “default” action or state that can be used or attained without spending the deliberation effort, and that there is a clear relationship between the time spent in deliberation and the quantifiable quality of the result. In the MAGNET case, the alternative to deliberation is to do nothing.

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<sup>6</sup> Under the “free disposal” assumption, the goal is to maximize revenue even if this means failing to allocate all the items at auction.

## 5 Conclusions

We have examined the problem of rational economic agents who must negotiate among themselves in a market environment in order to acquire the resources needed to accomplish their goals. We are interested in agents that are self-interested and heterogeneous, and we assume that a plan to achieve an agent's goal may be described in the form of a *task network*, containing task descriptions, precedence relationships among tasks, and time limits for individual tasks. Negotiation among agents is carried out by holding combinatorial reverse auctions in a marketplace, in which a *customer* agent offers a task network in the form of a *request for quotes* (RFQ). *Supplier* agents may then place bids on portions of the task network, each bid specifying the tasks they are interested in undertaking, durations and time limits for those tasks, and a price for the bid as a whole. The presence of temporal and precedence constraints among the items at auction requires extensions to the standard winner-determination procedures for combinatorial auctions, and the use of the enhanced winner-determination procedure within the context of a real-time negotiation requires us to be able to predict its runtime when planning the negotiation process.

There are a number of real-world business scenarios where such a capability would be of value. These include flexible manufacturing, mass customization, travel arrangement, logistics and international shipping, health care resource management, and large-scale systems management. Each of these areas is characterized by limited capabilities and suboptimal performance, due at least in part to the limits imposed by human problem-solving capabilities. In each of these areas, a general ability to coordinate plans among multiple independent suppliers would be of benefit, but does not exist or is not used effectively because of an inability to solve the resulting combinatorial problems. The use of extended combinatorial auctions such as we propose is one approach to solving these problems. There are many difficulties yet to be overcome before this vision can be realized, however, not the least of which is that such auction-based markets would not be effective without wide adoption of new technology across an industry, and a willingness to delegate at least some level of autonomy and authority to that new technology.

We have designed and implemented a testbed, which we call MAGNET for Multi-AGent NEgotiation Testbed, to begin exploring and testing this vision. It includes a customer agent, a rudimentary market infrastructure, and a simple simulation of a population of supplier agents. The customer agent implementation is designed so that virtually all behaviors can be specified and implemented in terms of responses to events. Events can be external occurrences, internal state changes, or the arrival of a particular point in time. The MAGNET software package is available to the research community under an open-source license.

When a goal arises, the agent and its principal must develop a plan, in the form of a *task network*. Once a plan is available, a *bid-process plan* must be developed to guide the negotiation process. The bid-process plan specifies which tasks are to be offered in which markets, allocates time to the bidding process and to the plan execution, and may split the bidding into phases in

order to mitigate risk. For each bidding step in the bid-process plan, time must be allocated to the customer to compose its RFQ, to the supplier to compose bids, and to the customer to evaluate bids. For each auction episode specified in the bid-process plan, a RFQ must be composed. The RFQ specifies a subset of tasks in the task network, and for each task, it specifies a *time window* within which that task must be accomplished. The setting of time windows is critical, because it influences the likelihood that bidders will bid, the prices bidders are likely to charge, and the difficulty of the resulting winner-determination process. If the time windows specified in the RFQ allow task precedence relationships to be violated, then the winner-determination process will need to choose a set of bids that can be composed into a feasible schedule. Once the RFQ has been issued and bids received, the agent must determine winners. We have described an optimal algorithm for determining winners based on an IDA\* framework.

Much work remains to be done before the vision of the MAGNET project is fully realized. Some of that work, particularly with respect to the supplier agent and its decision processes, is already under way by other members of the team.

With respect to the customer agent, many of the decision processes outlined in Section 2 still need to be worked out and tested. The present work has resulted in models for the auction winner-determination problem and the time that must be allocated to it. For the remainder of the decisions, we need models that will maximize the expected utility of the agent or its principal. These include composing the plan, developing the bid-process plan, allocating time to the deliberation processes of the customer and suppliers, balancing negotiation time against plan execution time, setting the time windows in the RFQ, scheduling the work in preparation for awarding bids, and dealing with unexpected events during plan execution. Babanov et al. [5] have addressed the problem of setting time windows in the customer's RFQ.

The language we currently use for plans and bids treats tasks as simple atomic objects, without attributes. There are many real-world problems in which attributes are important, both for specifying tasks and for expressing offers in bids. Examples include colors, quantities, dimensions, and quality attributes. In addition, many real-world operations operate on a "flow" basis. This includes the wine-making example we used in Chapter 2, in which the precedence between filling bottles and applying labels would normally be applied bottle-by-bottle, and not at the batch level. In addition, the expressivity of our bidding language is limited. A number of proposals have been made for more expressive bidding languages in combinatorial auctions [25, 4]. Bidding can also be done with *oracles*, which are functions passed from bidder to customer that can be evaluated to produce bid conditions. Some features of a more expressive bidding language would likely have minimal impact on the winner-determination process (parameterized quality values, for example), while others, including the use of oracles, could require wholesale re-invention.

## References

1. Collins, J., Tsvetov, M., Mobasher, B., Gini, M.: MAGNET: A multi-agent contracting system for plan execution. In: Proc. of SIGMAN, AAAI Press (1998) 63–68
2. Collins, J., Bilot, C., Gini, M., Mobasher, B.: Mixed-initiative decision support in agent-based automated contracting. In: Proc. of the Fourth Int'l Conf. on Autonomous Agents. (2000) 247–254
3. Nisan, N.: Bidding and allocation in combinatorial auctions. Technical report, Institute of Computer Science, Hebrew University (2000)
4. Boutilier, C., Hoos, H.H.: Bidding languages for combinatorial auctions. In: Proc. of the 17th Joint Conf. on Artificial Intelligence. (2001) 1211–1217
5. Babanov, A., Collins, J., Gini, M.: Asking the right question: Risk and expectation in multi-agent contracting. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing* **17** (2003) 173–186
6. Collins, J., Jamison, S., Gini, M., Mobasher, B.: Temporal strategies in a multi-agent contracting protocol. In: AAAI-97 Workshop on AI in Electronic Commerce. (1997)
7. Hillier, F.S., Lieberman, G.J.: *Introduction to Operations Research*. McGraw-Hill (1990)
8. Parkes, D.C., Ungar, L.H.: An auction-based method for decentralized train scheduling. In: Proc. of the Fifth Int'l Conf. on Autonomous Agents, Montreal, Quebec, ACM Press (2001) 43–50
9. Collins, J., Gini, M.: An integer programming formulation of the bid evaluation problem for coordinated tasks. In Dietrich, B., Vohra, R.V., eds.: *Mathematics of the Internet: E-Auction and Markets*. Volume 127 of IMA Volumes in Mathematics and its Applications. Springer-Verlag, New York (2001) 59–74
10. Collins, J., Bilot, C., Gini, M., Mobasher, B.: Decision processes in agent-based automated contracting. *IEEE Internet Computing* **5** (2001) 61–72
11. Russell, S.J., Norvig, P.: *Artificial Intelligence: A Modern Approach*. Prentice-Hall, Upper Saddle River, NJ (1995)
12. Sandholm, T.: An algorithm for winner determination in combinatorial auctions. In: Proc. of the 16th Joint Conf. on Artificial Intelligence. (1999) 524–547
13. Sandholm, T.: Algorithm for optimal winner determination in combinatorial auctions. *Artificial Intelligence* **135** (2002) 1–54
14. Collins, J.: *Solving Combinatorial Auctions with Temporal Constraints in Economic Agents*. PhD thesis, University of Minnesota, Minneapolis, Minnesota (2002)
15. Korf, R.E.: Depth-first iterative deepening: An optimal admissible tree search. *Artificial Intelligence* **27** (1985) 97–109
16. Collins, J., Demir, G., Gini, M.: Bidtree ordering in IDA\* combinatorial auction winner-determination with side constraints. In Padget, J., Shehory, O., Parkes, D., Sadeh, N., Walsh, W., eds.: *Agent Mediated Electronic Commerce IV*. Volume LNAI2531., Springer-Verlag (2002) 17–33
17. Rosenschein, J.S., Zlotkin, G.: *Rules of Encounter*. MIT Press, Cambridge, MA (1994)
18. Sandholm, T.W., Lesser, V.: On automated contracting in multi-enterprise manufacturing. In: *Distributed Enterprise: Advanced Systems and Tools*, Edinburgh, Scotland (1995) 33–42
19. Sandholm, T.W.: *Negotiation Among Self-Interested Computationally Limited Agents*. PhD thesis, Department of Computer Science, University of Massachusetts at Amherst (1996)

20. Pollack, M.E.: Planning in dynamic environments: The DIPART system. In Tate, A., ed.: *Advanced Planning Technology*. AAAI Press (1996)
21. Wilkins, D.E., Myers, K.L.: A multiagent planning architecture. In: *Proc. Int'l Conf. on AI Planning Systems*. (1998) 154–162
22. Wellman, M.P., Walsh, W.E., Wurman, P.R., MacKie-Mason, J.K.: Auction protocols for decentralized scheduling. *Games and Economic Behavior* **35** (2001) 271–303
23. Varian, H.R., MacKie-Mason, J.K.: Generalized vickrey auctions. Technical report, University of Michigan (1995)
24. Walsh, W.E., Wellman, M., Ygge, F.: Combinatorial auctions for supply chain formation. In: *Proc. of ACM Conf on Electronic Commerce (EC'00)*. (2000) 260–269
25. Nisan, N.: Bidding and allocation in combinatorial auctions. In: *Proc. of ACM Conf on Electronic Commerce (EC'00)*, Minneapolis, Minnesota, ACM SIGecom, ACM Press (2000) 1–12
26. Bakos, Y.: The emerging role of electronic marketplaces on the Internet. *Comm. of the ACM* **41** (1998) 33–42
27. Chavez, A., Maes, P.: Kasbah: An agent marketplace for buying and selling goods. In: *Proc. of the First Int'l Conf. on the Practical Application of Intelligent Agents and Multi-Agent Technology*, London, UK, Practical Application Company (1996) 75–90
28. Rodriguez, J.A., Noriega, P., Sierra, C., Padget, J.: FM96.5 - A Java-based electronic auction house. In: *Second Int'l Conf on The Practical Application of Intelligent Agents and Multi-Agent Technology (PAAM'97)*, London (1997) 207–224
29. Sycara, K., Pannu, A.S.: The RETSINA multiagent system: towards integrating planning, execution, and information gathering. In: *Proc. of the Second Int'l Conf. on Autonomous Agents*. (1998) 350–351
30. Wellman, M.P., Wurman, P.R.: Market-aware agents for a multiagent world. *Robotics and Autonomous Systems* **24** (1998) 115–125
31. Tsvetovaty, M., Gini, M., Mobasher, B., Wieckowski, Z.: MAGMA: An agent-based virtual market for electronic commerce. *Journal of Applied Artificial Intelligence* **11** (1997) 501–524
32. Faratin, P., Sierra, C., Jennings, N.R.: Negotiation decision functions for autonomous agents. *Int. Journal of Robotics and Autonomous Systems* **24** (1997) 159–182
33. Wurman, P.R., Wellman, M.P., Walsh, W.E.: The Michigan Internet AuctionBot: A configurable auction server for human and software agents. In: *Second Int'l Conf. on Autonomous Agents*. (1998) 301–308
34. Collins, J., Arunachalam, R., Sadeh, N., Ericsson, J., Finne, N., Janson, S.: The supply chain management game for the 2005 trading agent competition. Technical Report CMU-ISRI-04-139, Carnegie Mellon University, Pittsburgh, PA 15213 (2004)
35. Stone, P., Schnapire, R.E., Csirik, M.L.L.J.A., McAllester, D.: ATTac-2001: A learning, autonomous bidding agent. Submitted to the Eighteenth National Conference on Artificial Intelligence (AAAI-2002) (2002)
36. Sycara, K., Decker, K., Williamson, M.: Middle-agents for the Internet. In: *Proc. of the 15th Joint Conf. on Artificial Intelligence*. (1997) 578–583
37. Sycara, K., Klusch, M., Widoff, S., Lu, J.: Dynamic service matchmaking among agents in open information environments. *SIGMOD Record (ACM Special Interests Group on Management of Data)* **28** (1999) 47–53

38. Fox, M.S.: An organization ontology for enterprise modeling: Preliminary concepts. *Computers in Industry* **19** (1996) 123–134
39. Gruninger, M., Fox, M.S.: An activity ontology for enterprise modelling. In: *Workshop on Enabling Technologies - Infrastructures for Collaborative Enterprises*, West Virginia University (1994)
40. Schlenoff, C., Ivester, R., Knutilla, A.: A robust process ontology for manufacturing systems integration. National Institute of Standards and Technology (1998)
41. McAfee, R., McMillan, P.J.: Auctions and bidding. *Journal of Economic Literature* **25** (1987) 699–738
42. de Vries, S., Vohra, R.: Combinatorial auctions: a survey. Technical report, Technische Universität München (2001)
43. Rothkopf, M.H., Pekeč, A., Harstad, R.M.: Computationally manageable combinatorial auctions. *Management Science* **44** (1998) 1131–1147
44. Sandholm, T., Suri, S.: Bob: Improved winner determination in combinatorial auctions and generalizations. *Artificial Intelligence* **145** (2003) 33–58
45. Hoos, H.H., Boutilier, C.: Solving combinatorial auctions using stochastic local search. In: *Proc. of the Seventeen Nat'l Conf. on Artificial Intelligence*. (2000) 22–29
46. Andersson, A., Tenhunen, M., Ygge, F.: Integer programming for combinatorial auction winner determination. In: *Proc. of 4th Int'l Conf on Multi-Agent Systems*. (2000) 39–46
47. Parkes, D.C., Ungar, L.H.: Iterative combinatorial auctions: Theory and practice. In: *Proc. of the Seventeen Nat'l Conf. on Artificial Intelligence*. (2000) 74–81
48. Leyton-Brown, K., Shoham, Y., Tennenholtz, M.: An algorithm for multi-unit combinatorial auctions. In: *Proc. of the Seventeen Nat'l Conf. on Artificial Intelligence*, Austin, Texas (2000)
49. Greenwald, L., Dean, T.: Anticipating computational demands when solving time-critical decision-making problems. In Goldberg, K., Halperin, D., Latombe, J., Wilson, R., eds.: *The Algorithmic Foundations of Robotics*. A. K. Peters, Boston, MA (1995)
50. Boddy, M., Dean, T.: Decision-theoretic deliberation scheduling for problem solving in time-constrained environments. *Artificial Intelligence* **67** (1994) 245–286